RUHR UNIVERSITÄT BOCHUM

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Präsenzübungen zur Vorlesung

Quantenalgorithmen

WS 2013/2014

Blatt 2 / 7 November, 2013

Exercise 1:
Measure the first qubit of the states given below. What are the resulting states?

1. $|z_1\rangle = \frac{1+\sqrt{6}}{2\sqrt{6}}|00\rangle + \frac{1-\sqrt{6}}{2\sqrt{6}}|01\rangle + \frac{\sqrt{2}-\sqrt{3}}{2\sqrt{6}}|10\rangle + \frac{\sqrt{2}+\sqrt{3}}{2\sqrt{6}}|11\rangle$
2. $|z_2\rangle = \sqrt{\frac{1}{11}}|00\rangle + \sqrt{\frac{5}{11}}|01\rangle\sqrt{\frac{2}{11}}|20\rangle + \sqrt{\frac{3}{11}}|11\rangle$

Exercise 2:
Let $A \in \mathbb{C}^{m\times m}, B \in \mathbb{C}^{n\times n}$ be unitary matrices. Show that $A \otimes B \in \mathbb{C}^{nm\times nm}$ is also unitary.

Exercise 3:
For each two-qubit state below either express it as a product of two one-qubit states or show that the qubits are entangled:

1. $|z_1\rangle = \frac{9}{25}|00\rangle + \frac{12}{25}|01\rangle + \frac{12}{25}|10\rangle + \frac{16}{25}|11\rangle$
2. $|z_2\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |00\rangle + |11\rangle)$

Exercise 4:
Let $H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \in \mathbb{C}^{2\times 2}$. Define $C_H \in \mathbb{C}^{4\times 4}$ as 2-qubit unitary transformation such that $\forall b \in \{0,1\}$:

$$C_H|0b\rangle = |0\rangle \otimes H|b\rangle,\ C_H|1b\rangle = |1b\rangle.$$

1. Write out C_H explicitly;
2. Prove that for any quantum state $|q\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$ the outcomes of the following two processes (a) and (b) are the same:
   (a) we measure the first qubit of the state q and if the result is 0, we apply H to the second qubit, otherwise we do nothing;
   (b) we apply C_H to q and then measure the first qubit.

Exercise 5:
Consider the state

$$|z\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle).$$

Suppose all three qubits of this state are measured in the $\{|+\rangle, |-\rangle\}$ basis (i.e. $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$). What are the possible outcomes of these measurements? With what probabilities do they occure? Suppose the first qubit is measured in the $\{|+\rangle, |-\rangle\}$ basis and the second and the third qubits in the $\{|+I\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle), |-I\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)\}$ basis. Again, what are the possible outcomes of the measurements? With what probabilities fo they occure?