RUHR UNIVERSITÄT BOCHUM RUB

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Präsenzübungen zur Vorlesung

Quantenalgorithmen

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Blatt 1 /

Exercise 1:
You are given the qubit

$$|z\rangle = (\tfrac{1}{2} + \tfrac{i}{2})|0\rangle + (\tfrac{1}{2} - \tfrac{i}{2})|1\rangle.$$

Measure the qubit in computational basis: $\{|0\rangle, |1\rangle\}$. What are the probabilities of obtaining each of the two outcomes for this measurement? Show that $\langle z|z\rangle = 1$.

Exercise 2:
Unitary operators.

1. Show that the following operators (Pauli operators) are unitary:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad Z = \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$$

2. Apply transformation $Y(X|z\rangle)$ to $|z\rangle = \frac{i}{\sqrt{3}}|0\rangle + \frac{\sqrt{2}}{\sqrt{3}}|1\rangle$.
3. Show that $Y \cdot Z$ is unitary.

Exercise 3:
Show that $|\ \alpha_0|0\rangle + \alpha_1|1\rangle\ | = 1 \Leftrightarrow |\alpha_0|^2 + |\alpha_1|^2 = 1$.

Exercise 4:
Let $|x\rangle = \big(\frac{i\sqrt{2}}{2}; \frac{1}{2}; i; \frac{\sqrt{3}}{2}\big)$, $|y\rangle = \big(2; 4i\big)$, $|v\rangle = \big(-1; 0; \frac{1}{2}; 1\big)$, $|z\rangle = \big(-i; 0\big)$. Evaluate

1. $|x\rangle \otimes |y\rangle$
2. $\langle |v\rangle \otimes |y\rangle \,|\, |z\rangle \otimes |x\rangle\rangle$

Show that $\langle |v\rangle \otimes |y\rangle \,|\, |z\rangle \otimes |x\rangle\rangle = \langle v|x\rangle \cdot \langle y|z\rangle$.

Exercise 5:
Show that

$$|+\rangle = \tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle$$
$$|-\rangle = \tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle$$

is an orthonormal basis (a.k.a Hadamard basis). Measure $|\Psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$ in this basis.